Recitation 2. March 2

Focus:** LU **and** LDU **factorizations, taking inverses, symmetric matrices, column spaces

The LU factorization of a matrix A is the unique way of writing it:

$$\boxed{A = LU}$$

where L is a lower triangular matrix with 1's on the diagonal and U is in row echelon form. If A is square, then U is also square, in which case "row echelon form" means the same thing as "upper triangular". You can also write:

$$\boxed{A = LDU}$$

where both L and U have 1's on the diagonal, and D is diagonal. The discussion above works for almost all matrices A, and for those where it doesn't work, you can always write:

$$\boxed{PA = LDU}$$

for a suitable permutation matrix P.

The inverse of a square matrix A is the unique square matrix A with the property that $AA^{-1} = A^{-1}A = I$. One way to compute the inverse is to do Gauss-Jordan elimination on the augmented matrix $\left[\begin{array}{c|c} A & I \end{array}\right]$.

A symmetric matrix is one which is equal to its own **transpose**, i.e. its reflection across the diagonal.

The column space of a matrix is the vector space spanned by its columns.

1. Compute the $PA = LDU$ factorization of the matrix:

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$$

Solution:

2. Compute the inverse of the matrix:

$$A = \begin{bmatrix} 1 & 6 & -1 \\ 3 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

by Gauss-Jordan elimination on the augmented matrix $\left[\begin{array}{c|c} A & I \end{array}\right]$.

Solution:

3. Show that for any matrix A, the square matrix $S = A^T A$ is symmetric. For any vector $\boldsymbol{v}$, show that:

$$\boldsymbol{v}^T S \boldsymbol{v} \tag{1}$$

is a (1×1 matrix whose only entry is a) non-negative number.

Solution:

4. Find numbers a, b such that the column space of the matrix:

$$A = \begin{bmatrix} 1 & a \\ b & 3 \\ 2 & 1 \end{bmatrix}$$

is the plane in xyz space determined by the equation $2x + y - 3z = 0$.

Solution: